

SUPERSYMMETRY IN THERMO FIELD DYNAMICS

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Abstract

By considering the enlarged thermal system including the heat bath, it is shown that this system has supersymmetry which is not broken at finite temperature. The superalgebra is constructed and the Hamiltonian is expressed as the anti-commutator of two kinds of super charges. With this Hamiltonian and the thermal vacuum $|0(\beta)\rangle$, this supersymmetry is found to be preserved.

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I. Introduction

Supersymmetry at finite temperature has been studied in detail by Girardello et.al (1981), Van Hove (1982), Das (1989), Witten (1982), Das and Kaku (1978), Teshima (1983), Umezawa, Matsumoto and Tachiki (1982), and Umezawa (1993). Nevertheless, the issue of whether supersymmetry is broken or not, at finite temperature has raised some controversy. Girardello et.al (1981) argued that supersymmetry (SUSY) is broken at positive temperature even when unbroken at $T = 0$. In response to this, Van Hove (1982), suggested that when a change of an operator under SUSY transformation at finite temperature is considered, one should take into account the Klein operator. When this operator is incorporated, Van Hove (1982) shows that the thermal average of this change of an operator is zero for all T , thereby maintaining SUSY at finite temperature. On the other hand, Das (1989) has considered this issue within the context of the real time formalism or Thermo Field Dynamics (TFD) of Umezawa (Umezawa et.al (1982), Takahashi and Umezawa (1996)) and concluded that SUSY is broken at finite temperature, by evaluating the statistical average of the SUSY Hamiltonian at $T = 0$ as its vacuum expectation value in the 'thermal vacuum' $|0(\beta)\rangle$ (where $\beta = 1/kT$, k being the Boltzmann constant) and showing that it is non-zero at finite temperature.

It is the purpose of this work to examine first the construction of SUSY algebra in TFD and then use it to understand SUSY at finite temperature by enlarging the Fock space including that of the tilde operators. This procedure is analogous to the treatment of \hat{G} -symmetry by Umezawa and will be explained later. We will exhibit mathematical possibilities of preserving supersymmetry at finite temperature in agreement with Van Hove.

II. Brief Outline of Thermo Field Dynamics

In thermofield dynamics (Umezawa et.al (1982), Umezawa (1993) and Takahashi and Umezawa (1996)), the 'thermal vacuum' expectation value of an operator is equated to its statistical average. The 'thermal vacuum' is temperature dependent in a doubled Fock space. This construction procedure leads to the introduction of 'tilde' operators and the doubled Fock space is a direct product of the two Fock spaces for non-tilde and tilde creation and

annihilation operators. This doubling nature is one of the most fundamental and universal feature of all of the thermal quantum field formalism. For an ensemble of free Bosons with frequency ω , one has the Hamiltonian

$$H_B = \omega a^\dagger a,$$

$$[a, a^\dagger] = 1, \quad (1)$$

where a and a^\dagger are the annihilation and creation operators for Bosons. One introduces the tilde fields by the Hamiltonian

$$\tilde{H}_B = \omega \tilde{a}^\dagger \tilde{a},$$

$$[\tilde{a}, \tilde{a}^\dagger] = 1,$$

$$[a, \tilde{a}] = [a, \tilde{a}^\dagger] = 0, \quad (2)$$

where \tilde{a} and \tilde{a}^\dagger are the annihilation and creation operators for the tilde Bosonic fields. The thermal vacuum is given by

$$|0(\beta)\rangle = (1 - \exp(-\beta\omega))^{\frac{1}{2}} \exp(e^{-\beta\omega/2} a^\dagger \tilde{a}^\dagger) |0, \tilde{0}\rangle, \quad (3)$$

where a doubling of Fock space is exhibited. The operators $a, a^\dagger, \tilde{a}, \tilde{a}^\dagger$ pertain to zero temperature. The corresponding operators at finite temperature, namely, $a(\beta), a^\dagger(\beta), \tilde{a}(\beta), \tilde{a}^\dagger(\beta)$ are obtained from $a, a^\dagger, \tilde{a}, \tilde{a}^\dagger$ by Bogoliubov transformation. It is important to note that while

$$a |0(\beta)\rangle \neq 0, \quad (4)$$

we have

$$a(\beta) |0(\beta)\rangle = 0, \quad (5)$$

so that the Fock space at finite temperature is spanned by

$$|0(\beta)\rangle, a^\dagger(\beta) |0(\beta)\rangle, \tilde{a}^\dagger(\beta) |0(\beta)\rangle, \frac{1}{\sqrt{n!}} \frac{1}{\sqrt{m!}} (a^\dagger(\beta))^n (\tilde{a}^\dagger(\beta))^m |0(\beta)\rangle. \quad (6)$$

For an ensemble of Fermions of frequency ω (say), one has similar relations with the commutator replaced by anti-commutator, and the Fock space at finite temperature will be spanned by

$$|0(\beta)\rangle, f^\dagger(\beta)|0(\beta)\rangle, \tilde{f}^\dagger(\beta)|0(\beta)\rangle, f^\dagger(\beta)\tilde{f}^\dagger(\beta)|0(\beta)\rangle. \quad (7)$$

A physical interpretation of the doubling of the degrees of freedom, namely, a, a^\dagger and $\tilde{a}, \tilde{a}^\dagger$ for Bosons and/or f, f^\dagger and $\tilde{f}, \tilde{f}^\dagger$ for Fermions is the following. When the vacuum $|0(\beta)\rangle$ is required to be independent of time, as it should be, we choose the total Hamiltonian for free Boson fields in TFD as

$$\hat{H}_B = \int d^3k \omega_k \{a_k^\dagger a_k - \tilde{a}_k^\dagger \tilde{a}_k\}. \quad (8)$$

To an external stimulus, at $T \neq 0$, certain number of quantum particles are condensed in this system, which is in thermal equilibrium state with temperature T . The absorption of the external energy by the system results in (1) absorption by excitation of additional quanta and (2) excitation of quantum present in the vacuum, creating a hole. This is how one has doubling in TFD. In fact, Umezawa, Matsumoto and Tachiki (1982) attribute the excitation of holes to that in the thermal reservoir.

In studying the properties of dynamical observables, it is expected to use the non-tilde operators. However, in studying the symmetry properties of the system, one needs both the tilde and non-tilde operators. This is emphasised in Umezawa (1993), in studying the spontaneous breakdown of \hat{G} symmetry (that of the Bogoliubov transformation). In this Reference, the \hat{G} symmetry is defined to be spontaneously broken when $\hat{G}|0(\beta)\rangle \neq 0$ while $[\hat{G}, \hat{H}_0] = 0$. It is to be noted that the hat-Hamiltonian is used which has the tilde operators as well (see sections 7.2.4 and 7.3). For a system of free Fermions, the total Hamiltonian (analogue of (8)) is

$$\hat{H}_F = \int d^3k \omega_k \{f_k^\dagger f_k - \tilde{f}_k^\dagger \tilde{f}_k\} \quad (9)$$

with the (Fermion) Fock space in (7).

III. Supersymmetric Algebra

Following Van Hove (1982), we expect supersymmetry to have non-trivial consequences not only at $T = 0$ but also at finite temperature, since all

excited states must also some how reflect the supersymmetry property. In view of this we wish to examine *the possibility of maintaining supersymmetry at finite temperature as well*. By considering the enlarged Fock space (6) and (7), and using (8) and (9), we will arrive at this possibility, agreeing with Van Hove (1982).

We demonstrate this by considering a system of free Bosons and free Fermions. At zero temperature, the Bosons are described by the creation and annihilation operators a^\dagger and a and the corresponding tilde operators satisfying the algebra

$$[a, a^\dagger] = 1 \quad ; \quad [a, a] = 0,$$

$$[\tilde{a}, \tilde{a}^\dagger] = 1, \quad [\tilde{a}, \tilde{a}] = 0 \quad ; \quad [a, \tilde{a}] = 0, \quad [a, \tilde{a}^\dagger] = 0, \quad (10)$$

and similarly the Fermions are described by $f, f^\dagger, \tilde{f}, \tilde{f}^\dagger$ satisfying the algebra

$$\begin{aligned} \{f, f^\dagger\} &= 1 \quad , \quad f^2 = (f^\dagger)^2 = 0, \\ \{\tilde{f}, \tilde{f}^\dagger\} &= 1 \quad , \quad \tilde{f}^2 = (\tilde{f}^\dagger)^2 = 0, \\ \{f, \tilde{f}\} &= \{f, \tilde{f}^\dagger\} = 0. \end{aligned} \quad (11)$$

We construct Fermionic(super) charge operators (generators of Supersymmetry) as

$$\begin{aligned} Q_+ &= af^\dagger \quad ; \quad Q_- = a^\dagger f, \\ q_+ &= \tilde{a}\tilde{f}^\dagger \quad ; \quad q_- = \tilde{a}^\dagger \tilde{f}. \end{aligned} \quad (12)$$

These operators are *nilpotent*, namely, $Q_+^2 = Q_-^2 = q_+^2 = q_-^2 = 0$ and convert boson to fermion and vice-versa when acting on the representative state $|n_B, \tilde{n}_B, n_F, \tilde{n}_F\rangle$. The elements of the superalgebra are

$$Q_\pm, q_\pm, (N_B + N_F), (\tilde{N}_B + \tilde{N}_F), \quad (13)$$

where $N_B = a^\dagger a$, $N_F = f^\dagger f$, $\tilde{N}_B = \tilde{a}^\dagger \tilde{a}$, $\tilde{N}_F = \tilde{f}^\dagger \tilde{f}$. This algebra is closed

$$\begin{aligned} \{Q_+, Q_-\} &= N_B + N_F, \\ \{q_+, q_-\} &= \tilde{N}_B + \tilde{N}_F, \\ \{Q_+, q_+\} &= \{Q_-, q_-\} = \{Q_+, q_-\} = \{Q_-, q_+\} = 0, \end{aligned}$$

$$[Q_{\pm}, (N_B + N_F)] = 0,$$

$$[Q_{\pm}, (\tilde{N}_B + \tilde{N}_F)] = 0,$$

$$[q_{\pm}, (N_B + N_F)] = 0,$$

$$[q_{\pm}, (\tilde{N}_B + \tilde{N}_F)] = 0, \quad (14)$$

satisfying the structure $\{O, O\} = E$, $[O, E] = E$, $[E, E] = E$ for even (E) and odd (O) operators. The total Hamiltonian for supersymmetric oscillator $\hat{H} = a^\dagger a - \tilde{a}^\dagger \tilde{a} + f^\dagger f - \tilde{f}^\dagger \tilde{f}$ is given by the anti-commutator

$$\hat{H} = \{Q_+, Q_-\} - \{q_+, q_-\}, \quad (15)$$

and Q_{\pm}, q_{\pm} are Fermionic constants of motion, i.e.,

$$[Q_{\pm}, \hat{H}] = [q_{\pm}, \hat{H}] = 0. \quad (16)$$

The supersymmetric vacuum state at zero temperature is $|0\rangle = |0_B, \tilde{0}_B, 0_F, \tilde{0}_F\rangle$ and since this vacuum is annihilated by $a, \tilde{a}, f, \tilde{f}$, it follows that

$$\langle 0 | \hat{H} | 0 \rangle = 0, \quad (17)$$

and

$$\begin{aligned} Q_{\pm} | 0 \rangle &= 0, \\ q_{\pm} | 0 \rangle &= 0. \end{aligned} \quad (18)$$

Thus the supersymmetry constructed in (13) and (14) is not broken at zero temperature.

IV. Supersymmetry at finite temperature

At finite temperature, we will exhibit three mathematical possibilities to examine whether supersymmetry is broken or not. In view of the structure

of vacua at finite temperature in (6) and (7), we choose the thermal vacuum for the supersymmetric case as

$$|0(\beta)\rangle = |0_B(\beta), \tilde{0}_B(\beta), 0_F(\beta), \tilde{0}_F(\beta)\rangle. \quad (19)$$

The zero-temperature operators $a, \tilde{a}, f, \tilde{f}$ are related to the 'temperature dependent' operators $a(\beta), \tilde{a}(\beta), f(\beta), \tilde{f}(\beta)$ which annihilate the above 'thermal vacuum', by the (inverse) Bogoliubov transformation (Takahashi and Umezawa (1996))

$$\begin{aligned} a &= u(\beta)a(\beta) + v(\beta)\tilde{a}^\dagger(\beta), \\ \tilde{a} &= u(\beta)\tilde{a}(\beta) + v(\beta)a^\dagger(\beta), \\ f &= U(\beta)f(\beta) + V(\beta)\tilde{f}^\dagger(\beta), \\ \tilde{f} &= U(\beta)\tilde{f}(\beta) - V(\beta)f^\dagger(\beta), \end{aligned} \quad (20)$$

where

$$\begin{aligned} u(\beta) &= (1 - e^{-\beta\omega})^{-\frac{1}{2}}, \\ v(\beta) &= (e^{\beta\omega} - 1)^{-\frac{1}{2}}, \\ U(\beta) &= (1 + e^{-\beta\omega})^{-\frac{1}{2}}, \\ V(\beta) &= (1 + e^{\beta\omega})^{-\frac{1}{2}}. \end{aligned} \quad (21)$$

Method.1

In this method, the thermal vacuum is given by (19) and the Hamiltonian by (15) (involving zero temperature operators). Since now we have $a |0(\beta)\rangle \neq 0, \tilde{a} |0(\beta)\rangle \neq 0, f |0(\beta)\rangle \neq 0, \tilde{f} |0(\beta)\rangle \neq 0$, we need to use the Bogoliubov transformation (20).

Then it follows

$$a |0(\beta)\rangle = v(\beta)\tilde{a}^\dagger(\beta) |0(\beta)\rangle,$$

so that,

$$\begin{aligned} \langle 0(\beta) | a^\dagger a | 0(\beta) \rangle &= v^2(\beta) \langle 0(\beta) | \tilde{a}(\beta)\tilde{a}^\dagger(\beta) | 0(\beta) \rangle \\ &= v^2(\beta) \langle 0(\beta) | 1 + \tilde{a}^\dagger(\beta)\tilde{a}(\beta) | 0(\beta) \rangle = v^2(\beta), \end{aligned}$$

and

$$\tilde{a} | 0(\beta) > = v(\beta) a^\dagger(\beta) | 0(\beta) >,$$

so that

$$< 0(\beta) | \tilde{a}^\dagger \tilde{a} | 0(\beta) > = v^2(\beta) < 0(\beta) | a(\beta) a^\dagger(\beta) | 0(\beta) > = v^2(\beta).$$

Similarly, we have from

$$\begin{aligned} f | 0(\beta) > &= V(\beta) \tilde{f}^\dagger(\beta) | 0(\beta) >, \\ \tilde{f} | 0(\beta) > &= -V(\beta) f^\dagger(\beta) | 0(\beta) >, \end{aligned}$$

so that

$$\begin{aligned} < 0(\beta) | f^\dagger f | 0(\beta) > &= V^2(\beta), \\ < 0(\beta) | \tilde{f}^\dagger \tilde{f} | 0(\beta) > &= V^2(\beta). \end{aligned}$$

It then follows from these that

$$\begin{aligned} < 0(\beta) | \hat{H} | 0(\beta) > &= < 0(\beta) | a^\dagger a - \tilde{a}^\dagger \tilde{a} + f^\dagger f - \tilde{f}^\dagger \tilde{f} | 0(\beta) > \\ &= 0, \end{aligned} \tag{22}$$

implying that \hat{H} is invariant under supersymmetry. On the other hand,

$$\begin{aligned} Q_\pm | 0(\beta) > &\neq 0, \\ q_\pm | 0(\beta) > &\neq 0. \end{aligned} \tag{23}$$

Thus we realize a situation: *while the total Hamiltonian is supersymmetric invariant, the thermal vacuum is not. We realize, "spontaneous breakdown of supersymmetry"*. This is in contrast to the result of Das (1989) in which both his Hamiltonian and the thermal vacuum are not invariant - a case of explicit breaking of supersymmetry.

Method.2

We first motivate the Method.2, by realizing that in the expressions $< 0(\beta) | \hat{H} | 0(\beta) >$ and those such as $Q_\pm | 0(\beta) >$, with \hat{H} given in (15) and Q_\pm, q_\pm in (12), the state vector $| 0(\beta) > = | 0_B(\beta), \tilde{0}_B(\beta), 0_F(\beta), \tilde{0}_F(\beta) >$ is

in the doubled Fock space with the spectrum $| n_B(\beta), \tilde{n}_B(\beta), n_F(\beta), \tilde{n}_F(\beta) > \simeq ((a(\beta))^\dagger)^{n_B} ((\tilde{a}(\beta))^\dagger)^{\tilde{n}_B} ((f(\beta))^\dagger)^{n_F} ((\tilde{f}(\beta))^\dagger)^{\tilde{n}_F} | 0(\beta) >$. The states including the vacuum are temperature dependent. On the other hand, the operators \hat{H} and Q_\pm, q_\pm are in terms of temperature independent creation and annihilation operators. In view of this disparity, it is more appropriate to have operators also Bogoliubov transformed so that they act on the same Fock space for examining the symmetry properties of the total system at finite temperature and this is the reason for considering Method.2.

First. we consider the Bogoliubov transformed operators (Takahashi and Umezawa (1996)) which are temperature dependent, as

$$\begin{aligned} a(\beta) &= u(\beta)a - v(\beta)\tilde{a}^\dagger, \\ \tilde{a}(\beta) &= u(\beta)\tilde{a} - v(\beta)a^\dagger, \\ f(\beta) &= U(\beta)f - V(\beta)\tilde{f}^\dagger, \\ \tilde{f}(\beta) &= U(\beta)\tilde{f} + V(\beta)f^\dagger, \end{aligned} \tag{24}$$

which is the inverse of (20) and the functions $u(\beta), v(\beta), U(\beta), V(\beta)$ are given in (21). It can be verified that these temperature dependent operators satisfy the algebra

$$\begin{aligned} [a(\beta), a^\dagger(\beta)] &= 1, \\ [\tilde{a}(\beta), \tilde{a}^\dagger(\beta)] &= 1; [a(\beta), \tilde{a}(\beta)] = [a(\beta), \tilde{a}^\dagger(\beta)] = 0, \\ \{f(\beta), f^\dagger(\beta)\} &= 1 \quad ; \quad f^2(\beta) = (f^\dagger(\beta))^2 = 0, \\ \{\tilde{f}(\beta), \tilde{f}^\dagger(\beta)\} &= 1 \quad ; \quad \{f(\beta), \tilde{f}(\beta)\} = \{f(\beta), \tilde{f}^\dagger(\beta)\} = 0. \end{aligned} \tag{25}$$

With these, we introduce the temperature dependent super charges as

$$\begin{aligned} Q_+(\beta) &= a(\beta)f^\dagger(\beta) \quad ; \quad Q_-(\beta) = a^\dagger(\beta)f(\beta), \\ q_+(\beta) &= \tilde{a}(\beta)\tilde{f}^\dagger(\beta) \quad ; \quad q_-(\beta) = \tilde{a}^\dagger(\beta)\tilde{f}(\beta). \end{aligned} \tag{26}$$

They are nil-potent. With the number operators, $(N_B(\beta) + N_F(\beta)) ; (\tilde{N}_B(\beta) + \tilde{N}_F(\beta))$, where $N_B(\beta) = a^\dagger(\beta)a(\beta)$, $\tilde{N}_B = \tilde{a}^\dagger(\beta)\tilde{a}(\beta)$,

$N_F(\beta) = f^\dagger(\beta)f(\beta)$, $\tilde{N}_F(\beta) = \tilde{f}^\dagger(\beta)\tilde{f}(\beta)$, they form a closed super algebra, namely,

$$\begin{aligned}\{Q_+(\beta), Q_-(\beta)\} &= N_B(\beta) + N_F(\beta), \\ \{q_+(\beta), q_-(\beta)\} &= \tilde{N}_B(\beta) + \tilde{N}_F(\beta), \\ \{Q_+(\beta), q_+(\beta)\} &= \{Q_+(\beta), q_-(\beta)\} = 0, \\ \{Q_-(\beta), q_+(\beta)\} &= \{Q_-(\beta), q_-(\beta)\} = 0,\end{aligned}$$

$$[Q_\pm(\beta), (N_B(\beta) + N_F(\beta))] = 0,$$

$$[Q_\pm(\beta), (\tilde{N}_B(\beta) + \tilde{N}_F(\beta))] = 0,$$

$$[q_\pm(\beta), (N_B(\beta) + N_F(\beta))] = 0,$$

$$[q_\pm(\beta), (\tilde{N}_B(\beta) + \tilde{N}_F(\beta))] = 0. \quad (27)$$

Furthermore, we realize the important relation,

$$\begin{aligned}\{Q_+(\beta), Q_-(\beta)\} - \{q_+(\beta), q_-(\beta)\} &= a^\dagger(\beta)a(\beta) - \tilde{a}^\dagger(\beta)\tilde{a}(\beta) \\ &+ f^\dagger(\beta)f(\beta) - \tilde{f}^\dagger(\beta)\tilde{f}(\beta) \equiv \hat{H}(\beta).\end{aligned} \quad (28)$$

This important relation expresses the total Hamiltonian at finite temperature as anti-commutator of temperature dependent super charges and therefore $Q_\pm(\beta), q_\pm(\beta)$ commute with $\hat{H}(\beta)$ showing that they are the fermionic constants of motion.

It can be verified upon using (24) that $\hat{H}(\beta)$ in (28) is the same as \hat{H} in (15) showing the invariance of the total Hamiltonian under Bogoliubov transformation. Using, $a(\beta) | 0(\beta) \rangle = 0$; $\tilde{a}(\beta) | 0(\beta) \rangle = 0$; $f(\beta) | 0(\beta) \rangle = 0$; $\tilde{f}(\beta) | 0(\beta) \rangle = 0$, it follows that

$$\begin{aligned}\langle 0(\beta) | \hat{H}(\beta) | 0(\beta) \rangle &= 0, \\ Q_\pm(\beta) | 0(\beta) \rangle &= 0, \\ q_\pm(\beta) | 0(\beta) \rangle &= 0.\end{aligned} \quad (29)$$

Thus, both the total Hamiltonian $\hat{H}(\beta)$ and the thermal vacuum $|0(\beta)\rangle$ are invariant under supersymmetry and so supersymmetry is not broken at finite temperature, in agreement with Van Hove (1982).

Method.3

The construction of the supersymmetric charges Q_{\pm}, q_{\pm} in (12) and $Q_{\pm}(\beta), q_{\pm}(\beta)$ in (26) in examining the supersymmetry breaking or not, using the total Hamiltonian \hat{H} and $\hat{H}(\beta)$ is on mathematical grounds, in the sense that in thermo field dynamics, the observables are analysed in terms of non-tilde operators while the above mathematical procedure includes tilde operators as well. It is still possible to realize unbroken supersymmetry at $T \neq 0$ without using the tilde operators by restricting the super algebra to

$$Q_{\pm}(\beta) \quad , \quad (N_B(\beta) + N_F(\beta)). \quad (30)$$

This algebra is closed, namely,

$$\begin{aligned} \{Q_+(\beta), Q_-(\beta)\} &= N_B(\beta) + N_F(\beta), \\ [Q_{\pm}(\beta), (N_B(\beta) + N_F(\beta))] &= 0. \end{aligned} \quad (31)$$

The Hamiltonian of the system is identified with

$$H(\beta) = \{Q_+(\beta), Q_-(\beta)\} = a^\dagger(\beta)a(\beta) + f^\dagger(\beta)f(\beta), \quad (32)$$

so that

$$[Q_{\pm}(\beta), H(\beta)] = 0, \quad (33)$$

giving the fermionic constants of motion with respect to $H(\beta)$. The ground state is the thermal vacuum $|0(\beta)\rangle$ as before. Then it follows

$$\begin{aligned} \langle 0(\beta) | H(\beta) | 0(\beta) \rangle &= 0, \\ Q_{\pm}(\beta) | 0(\beta) \rangle &= 0, \end{aligned} \quad (34)$$

showing that supersymmetry is not broken at finite temperature. The situation at zero temperature in this case is obtained by taking the limit $\beta \rightarrow \infty$. We have from (20) and (21), $u(\beta) \rightarrow 1$; $v(\beta) \rightarrow 0$; $a(\beta) \rightarrow a$; $f(\beta) \rightarrow$

$f ; H(\beta) \rightarrow H ; |0(\beta)\rangle \rightarrow |0\rangle$ as $\beta \rightarrow \infty$ and then we recover the zero temperature case and this has supersymmetry unbroken.

V. Summary

We have examined the supersymmetric structure in thermo field dynamics by considering the enlarged Fock space. Supersymmetric generators are constructed and three methods, in which the Hamiltonian is governed by the anti-commutator of super charges, are studied. Besides realizing spontaneous breakdown of supersymmetry in Method.1, there are two possibilities to realize unbroken supersymmetry at finite temperature. These have well defined zero temperature limit in which the supersymmetry is not broken. These results are in agreement with Van Hove. This analysis goes through in the Lagrangian formulation in TFD and since this is straightforward, we have not included this.

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References

- Das A, Physica 1989 **A158** 1.
- Das A and Kaku M, 1978 Phys.Rev. **D18** 4540.
- Girardello L, Grisaru M T, and Salomonson N P, 1981 Nucl.Phys. **B178** 513.
- Takahashi Y and Umezawa H, 1996 Int.J.Mod.Phys. **B10** 1755.
- Teshima K, 1983 Phys.Lett. **B123** 226.
- Umezawa H, Matsumoto H, and Tachiki M 1982 *Thermo Field Dynamics*, North-Holland, Amsterdam.
- Umezawa H, 1993 *Advanced Field Theory*, AIP, New York.
- Van Hove L, 1982 Nucl.Phys. **B207** 15.
- Witten E, 1982 Nucl.Phys. **B202** 253.